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SHORT COMMUNICATION

DISCUSSION: 'MACROSCALE SURFACE ROUGHNESS AND FRICTIONAL RESISTANCE IN OVERLAND FLOW' BY D. S. L. LAWRENCE

ATHOL D. ABRAHAMS*
Department of Geography, State University of New York at Buffalo, Buffalo, NY 14261, USA

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ABSTRACT

Lawrence argued that the inundation ratio Λ, defined as the mean flow depth d divided by the roughness height k, is the dominant control of flow resistance f and should be used as the primary variable when evaluating the hydraulics of overland flow on rough surfaces. Lawrence defined three flow regimes on the basis of Λ and developed an expression for f in terms of Λ for each regime. Common sense, however, suggests that f is independent of Λ where Λ<1 because when roughness elements protrude through the flow, the value of f for the flow is the same regardless of the height of the elements. The error appears to have crept in as a result of Lawrence's representation of roughness elements by hemispheres. Lawrence found that $f \propto d/k$, which she interpreted to mean $f \propto \Lambda$. However, in her model the length dimension denoted by k is in fact half the breadth $b/2$ of the roughness elements. The distinction between k and $b/2$ is important, especially for roughness elements where $k \neq b/2$. Thus, contrary to Lawrence's claim, f is not generally a function of Λ. Instead, f is a function of Λ only where Λ>1. Where Λ<1, f is a function of $d/(b/2)$ or d/b. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: overland flow; resistance to flow; shallow flow hydraulics

Lawrence's (1997) recent paper on the resistance to overland flow is both stimulating and useful and represents an important contribution to the literature on this subject. The basic thesis of the paper is that the inundation ratio Λ, defined as the mean flow depth d divided by the roughness height k, is the dominant control of flow resistance, measured by the friction factor

$$\bar{f} = 8gSd / V^2 \tag{1}$$

where g is the acceleration of gravity, S is the energy gradient and V is the mean flow velocity, and that this ratio should be used as the primary variable when evaluating the hydraulics of overland flow on rough surfaces. Lawrence identified three flow regimes – the partial inundation regime where Λ<1, the marginal inundation regime where 1<Λ<10, and the well-inundated flow regime where Λ>10 – and developed an expression for f in terms of Λ for each regime. However, there is a flaw in Lawrence's thesis. Common sense suggests that contrary to Lawrence's claim, f is independent of Λ where Λ<1. Put simply, if roughness elements protrude through the flow, the value of f for the flow is the same regardless of the height of the elements and, hence, regardless of the value of Λ.

The error appears to have crept in as a result of Lawrence's representation of roughness elements by hemispheres. For such shapes where Λ<1 Lawrence deduced that

$$\bar{f} \approx \frac{8}{\pi} PC_D \,\mathrm{MIN}\left[\frac{\pi}{4}, \frac{d}{k}\right] \tag{2}$$

* Correspondence to: Prof. A. D. Abrahams, Department of Geography, State University of New York at Buffalo, Buffalo, NY 14261, USA

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where P is the proportion of the bed covered by the elements, C_D is the drag coefficient of the elements, and MIN[x,y] denotes the minimum value taken by the variables x and y. In Equation 2, $f \propto d/k$, which Lawrence interpreted to mean that $f \propto \Lambda$. However, scrutiny of the derivation of Equation 2 reveals that the length dimension denoted by k in this equation is in fact half the breadth $b/2$ of the roughness elements. Thus, f is actually a function of $d/(b/2)$ rather than Λ.

This conclusion is confirmed and the confusion between k and $b/2$ is removed if the roughness elements are assumed to be cylinders for which $k \neq b/2$. In Hirsch's (1996) flow resistance model (outlined by Abrahams *et al*. (1992)) the total form drag exerted by all the roughness elements in an area A_B is given by

$$\Sigma F_D = C_D \rho V^2 \Sigma A_F / 2 \tag{3}$$

where ρ is the fluid density and $\sum A_F$ is the sum of the inundated projected frontal areas of the elements. The related shear stress τ is then

$$\tau = \frac{C_D \rho V^2 \Sigma A_F}{2 A_B} \tag{4}$$

Noting from Equation 1 that

$$\tau = f V^2 g / 8 \tag{5}$$

and combining Equations 4 and 5, Hirsch obtained

$$f = 4 C_D \Sigma A_F / A_B \tag{6}$$

Now, for an area A_B covered with n cylinders of any height greater than d

$$\Sigma A_F = nbd \tag{7}$$

and

$$P = n\pi (b/2)^2 / A_B \tag{8}$$

so that

$$A_B = n\pi (b/2)^2 / P \tag{9}$$

Substituting Equations 7 and 9 into 6, one obtains

$$f = \frac{8}{\pi} C_D P \frac{d}{b/2} = \frac{16}{\pi} C_D P \frac{d}{b} \tag{10}$$

Disregarding the MIN[x,y] construction which does not apply to cylinders, this equation is the same as Equation 2 (i.e. Lawrence's model) except that it contains $b/2$ rather than k.

The distinction between k and $b/2$ is important, especially for roughness elements where $k \neq b/2$, such as occur in flume experiments with cylindrical roughness elements that are much taller than they are broad (e.g. Abrahams *et al*., 1998) or on grass-covered hillslopes or in grass-lined channels where the grass stems are much taller than they are thick (e.g. Petryk and Bosmajian, 1975). In all fairness to Lawrence, the field and laboratory data she used to evaluate her model are from studies that employed either stones or hemispheres for roughness elements. For such elements k is either equal or approximately equal to $b/2$, so her model fits the data reasonably well, and one can debate whether or not she should have distinguished between k and $b/2$. However, the fact remains that contrary to Lawrence's claim, f is not generally a function of Λ. Instead, f is a function of Λ only where $\Lambda > 1$. Where $\Lambda < 1$, f is a function of $d/(b/2)$ or d/b.

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